

Magnetic Materials

37-1 Understanding ferromagnetism

In this chapter we will discuss the behavior and peculiarities of ferromagnetic materials and of other strange magnetic materials. Before proceeding to study magnetic materials, however, we will review very quickly some of the things about the general theory of magnets that we learned in the last chapter.

First, we imagine the atomic currents inside the material that are responsible for the magnetism, and then describe them in terms of a volume current density $\mathbf{j}_{\text{mag}} = \nabla \times \mathbf{M}$. We emphasize that this is not supposed to represent the *actual* currents. When the magnetization is uniform the currents do not *really* cancel out precisely; that is, the whirling currents of one electron in one atom and the whirling currents of an electron in another atom do not overlap in such a way that the sum is exactly zero. Even within a single atom the distribution of magnetism is *not* smooth. For instance, in an iron atom the magnetization is distributed in a more or less spherical shell, not too close to the nucleus and not too far away. Thus, magnetism in matter is quite a complicated thing in its details; it is very irregular. However, we are obliged now to ignore this detailed complexity and discuss phenomena from a gross, average point of view. Then it is true that the *average* current in the interior region, over any finite area that is big compared with an atom, is zero when $\mathbf{M} = 0$. So, what we mean by magnetization per unit volume and \mathbf{j}_{mag} and so on, at the level we are now considering, is an average over regions that are large compared with the space occupied by a single atom.

In the last chapter, we also discovered that a ferromagnetic material has the following interesting property: above a certain temperature it is not strongly magnetic, whereas below this temperature it becomes magnetic. This fact is easily demonstrated. A piece of nickel wire at room temperature is attracted by a magnet. However, if we heat it above its Curie temperature with a gas flame, it becomes nonmagnetic and is not attracted toward the magnet—even when brought quite close to the magnet. If we let it lie near the magnet while it cools off, at the instant its temperature falls below the critical temperature it is suddenly attracted again by the magnet!

The general theory of ferromagnetism that we will use supposes that the spin of the electron is responsible for the magnetization. The electron has spin one-half and carries one Bohr magneton of magnetic moment $\mu = \mu_B = q_e \hbar / 2m$. The electron spin can be pointed either “up” or “down.” Because the electron has a negative charge, when its spin is “up” it has a *negative* moment, and when its spin is “down” it has a *positive* moment. With our usual conventions, the moment μ of the electron is opposite its spin. We have found that the energy of orientation of a magnetic dipole in a given applied field \mathbf{B} is $-\mu \cdot \mathbf{B}$, but the energy of the spinning electrons depends on the neighboring spin alignments as well. In iron, if the moment of a nearby atom is “up,” there is a very strong tendency that the moment of the one next to it will also be “up.” That is what makes iron, cobalt, and nickel so strongly magnetic—the moments all want to be parallel. The first question we have to discuss is *why*.

Soon after the development of quantum mechanics, it was noticed that there is a very strong *apparent* force—not a magnetic force or any other kind of actual force, but only an apparent force—trying to line the spins of nearby electrons *opposite* to one another. These forces are closely related to chemical valence forces. There is a principle in quantum mechanics—called the *exclusion principle*—that

37-1 Understanding ferromagnetism

37-2 Thermodynamic properties

37-3 The hysteresis curve

37-4 Ferromagnetic materials

37-5 Extraordinary magnetic materials

References: Bozorth, R. M., “Magnetism,” *Encyclopaedia Britannica*, Vol. 14, 1957, pp. 636–667.

Kittel, C., *Introduction to Solid State Physics*, John Wiley and Sons, Inc., New York, 2nd ed., 1956.

two electrons cannot occupy exactly the same state, that they cannot be in exactly the same condition as to location and spin orientation.* For example, if they are at the same point, the only alternative is to have their spins opposite. So, if there is a region of space between atoms where electrons like to congregate (as in a chemical bond) and we want to put another electron on top of one already there, the only way to do it is to have the spin of the second one pointed opposite to the spin of the first one. To have the spins parallel is against the law, unless the electrons stay away from each other. This has the effect that a pair of parallel-spin electrons near to each other have much more energy than a pair of opposite-spin electrons; the net effect is as though there were a force trying to turn the spin over. Sometimes this spin-turning force is called the *exchange force*, but that only makes it more mysterious—it is not a very good term. It is just because of the exclusion principle that electrons have a tendency to make their spins opposite. In fact, that is the explanation of the *lack* of magnetism in almost all substances! The spins of the free electrons on the outside of the atoms have tremendous tendency to balance in opposite directions. The problem is to explain why for materials like iron it is just the reverse of what we should expect.

We have summarized the supposed alignment effect by adding a suitable term in the energy equation, by saying that if the electron magnets in the neighborhood have a mean magnetization M , then the moment of an electron has a strong tendency to be in the same direction as the average magnetization of the atoms in the neighborhood. Thus, we may write for the two possible spin orientations,†

$$\begin{aligned} \text{Spin "up" energy} &= +\mu \left(H + \frac{\lambda M}{\epsilon_0 c^2} \right), \\ \text{Spin "down" energy} &= -\mu \left(H + \frac{\lambda M}{\epsilon_0 c^2} \right). \end{aligned} \tag{37.1}$$

When it was clear that quantum mechanics could supply a tremendous spin-orientating force—even if, apparently, of the wrong sign—it was suggested that ferromagnetism might have its origin in this same force, that due to the complexities of iron and the large number of electrons involved, the sign of the interaction energy would come out the other way around. Since the time this was thought of—in about 1927 when quantum mechanics was first being understood—many people have been making various estimates and semicalculations, trying to get a theoretical prediction for λ . The most recent calculations of the energy between the two electron spins in iron—assuming that the interaction is a direct one between the two electrons in neighboring atoms—still give the *wrong sign*. The present understanding of this is again to assume that the complexity of the situation is somehow responsible and to hope that the next man who makes the calculation with a more complicated situation will get the right answer!

It is believed that the up-spin of one of the electrons in the inside shell, which is making the magnetism, tends to make the conduction electrons which fly around the outside have the opposite spin. One might expect this to happen because the conduction electrons come into the same region as the “magnetic” electrons. Since they move around, they can carry their prejudice for being upside down over to the next atom; that is, one “magnetic” electron tries to force the conduction electrons to be opposite, and the conduction electron then makes the next “magnetic” electron opposite to *it*. The double interaction is equivalent to an interaction which tries to line up the two “magnetic” electrons. In other words, the tendency to make parallel spins is the result of an intermediary that tends to some extent to be opposite to both. This mechanism does not require that the conduction electrons be completely “upside down.” They could just have a slight prejudice to be down, just enough to load the “magnetic” odds the other way. This is the mechanism that

* See Chapter 43.

† We write these equations with $H = B - M/\epsilon_0 c^2$ instead of B to agree with the work of the last chapter. You might prefer to write $U = \pm \mu B_a = \pm \mu (B + \lambda' M/\epsilon_0 c^2)$, where $\lambda' = \lambda - 1$. It's the same thing.

the people who have calculated such things now believe is responsible for ferromagnetism. But we must emphasize that to this day nobody can calculate the magnitude of λ simply by knowing that the material is number 26 in the periodic table. In short, we don't thoroughly understand it.

Now let us continue with the theory, and then come back later to discuss a certain error involved in the way we have set it up. If the magnetic moment of a certain electron is "up," energy comes both from the external field and also from the tendency of the spins to be parallel. Since the energy is lower when the spins are parallel, the effect is sometimes thought of as due to an "effective internal field." But remember, it is *not* due to a true *magnetic* force; it is an interaction that is more complicated. In any case, we take Eqs. (37.1) as the formulas for the energies of the two spin states of a "magnetic" electron. At a temperature T , the relative probability of these two states is proportional to $e^{-\text{energy}/kT}$, which we can write as e^{+x} , with $x = \mu(H + \lambda M/\epsilon_0 c^2)/kT$. Then, if we calculate the mean value of the magnetic moment, we find (as in the last chapter) that it is

$$M = N\mu \tanh x. \quad (37.2)$$

Now we would like to calculate the internal energy of the material. We note that the energy of an electron is exactly proportional to the magnetic moment, so that the calculation of the mean moment and the calculation of the mean energy are the same—except that in place of μ in Eq. (37.2) we would write $-\mu B$, which is $-\mu(H + \lambda M/\epsilon_0 c^2)$. The mean energy is then

$$\langle U \rangle_{\text{av}} = -N\mu \left(H + \frac{\lambda M}{\epsilon_0 c^2} \right) \tanh x.$$

Now this is not quite correct. The term $\lambda M/\epsilon_0 c^2$ represents interactions of all possible *pairs* of atoms, and we must remember to count each pair only *once*. (When we consider the energy of one electron in the field of the rest and then the energy of a second electron in the field of the rest, we have counted part of the first energy once more.) Thus, we must divide the *mutual interaction term* by two, and our formula for the energy then turns out to be

$$\langle U \rangle_{\text{av}} = -N\mu \left(H + \frac{\lambda M}{2\epsilon_0 c^2} \right) \tanh x. \quad (37.3)$$

In the last chapter we discovered an interesting thing—that below a certain temperature the material finds a solution to the equations in which the magnetic moment is *not zero*, even with no external magnetizing field. When we set $H = 0$ in Eq. (37.2), we found that

$$\frac{M}{M_{\text{sat}}} = \tanh \left(\frac{T_c}{T} \frac{M}{M_{\text{sat}}} \right), \quad (37.4)$$

where $M_{\text{sat}} = N\mu$, and $T_c = \mu\lambda M_{\text{sat}}/k\epsilon_0 c^2$. When we solve this equation (graphically or otherwise), we find that the ratio M/M_{sat} as a function of T/T_c is a curve like that labeled "quantum theory" in Fig. 37-1. The dashed curve marked "cobalt, nickel" shows the experimental results for crystals of these elements. The theory and experiment are in reasonably good agreement. The figure also shows the result of the classical theory in which the calculation is carried out assuming that the atomic magnets can have all possible orientations in space. You can see that this assumption gives a prediction that is not even close to the experimental facts.

Even the quantum theory deviates from the observed behavior at both high and low temperatures. The reason for the deviations is that we have made a rather sloppy approximation in the theory: We have assumed that the energy of an atom depends upon the *mean* magnetization of its neighboring atoms. In other words, for each one that is "up" in the neighborhood of a given atom, there will be a contribution of energy due to that quantum mechanical alignment effect. But how many are there pointed "up"? On the average, that is measured by the

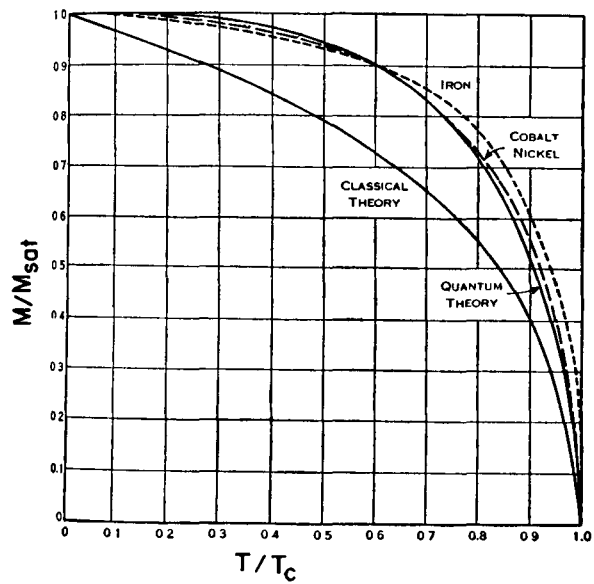


Fig. 37-1. The spontaneous magnetization ($H = 0$) of ferromagnetic crystals as a function of temperature. [Permission from *Encyclopaedia Britannica*.]

magnetization M —but only *on the average*. A particular atom somewhere might find *all* its neighbors “up.” Then its energy will be larger than the average. Another one might find some up and some down, perhaps averaging to zero, and it would have *no* energy from that term, and so on. What we ought to do is to use some more complicated kind of average, because the atoms in different places have different environments, and the numbers up and down are different for different ones. Instead of just taking one atom subjected to the average influence, we should take each one in its actual situation, compute its energy, and find the *average energy*. But how do we find out how many are “up” and how many are “down” in the neighborhood? That is, of course, just what we are trying to calculate—the number “up” and “down”—so we have a very complicated interconnected problem of correlations, a problem which has never been solved. It is an intriguing and exciting one which has existed for years and on which some of the greatest names in physics have written papers, but even they have not completely solved it.

It turns out that at low temperatures, when almost all the atomic magnets are “up” and only a few are “down,” it is easy to solve; and at high temperatures, far above the Curie temperature T_c when they are almost all random, it is again easy. It is often easy to calculate small departures from some simple, idealized situation, so it is fairly well understood why there are deviations from the simple theory at low temperature. It is also understood physically that for statistical reasons the magnetization *should* deviate at high temperatures. But the exact behavior near the Curie point has never been thoroughly figured out. That’s an interesting problem to work out some day if you want a problem that has never been solved.

37-2 Thermodynamic properties

In the last chapter we laid the groundwork necessary for calculating the thermodynamic properties of ferromagnetic materials. These are, naturally, related to the internal energy of the crystal, which includes interactions of the various spins, given by Eq. (37.3). For the energy of the spontaneous magnetization below the Curie point, we can set $H = 0$ in Eq. (37.3), and—noticing that $\tanh x = M/M_{\text{sat}}$ —we find a mean energy proportional to M^2 :

$$\langle U \rangle_{\text{av}} = - \frac{N\mu\lambda M^2}{2\epsilon_0 c^2 M_{\text{sat}}} \quad (37.5)$$

If we now plot the energy due to the magnetism as a function of temperature, we get a curve which is the negative of the square of the curve of Fig. 37-1, as drawn in Fig. 37-2(a). If we were to measure then the *specific heat* of such a material we would obtain a curve which is the derivative of 37-2(a). It is shown in Fig.

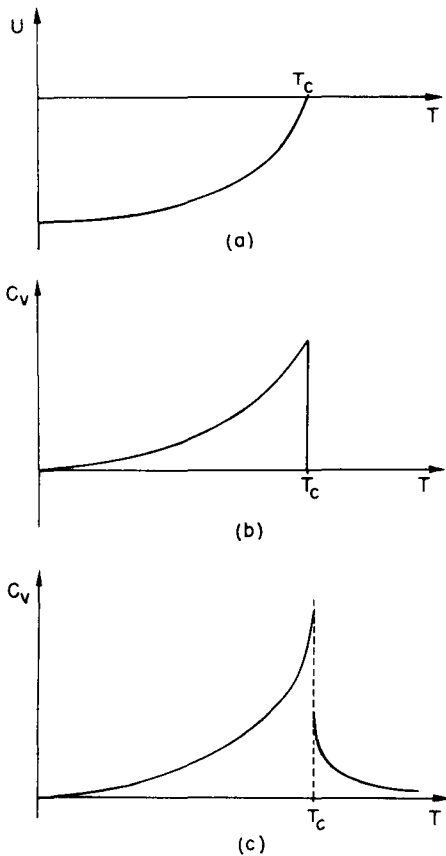


Fig. 37-2. The energy per unit volume and specific heat of a ferromagnetic crystal.

37-2(b). It rises slowly with increasing temperature, but falls suddenly to zero at $T = T_c$. The sharp drop is due to the change in slope of the magnetic energy and is reached right at the Curie point. So without any magnetic measurements at all we could have discovered that something was going on inside of iron or nickel by measuring this thermodynamic property. However, both experiment and improved theory (with fluctuations included) suggest that this simple curve is wrong and that the true situation is really more complicated. The curve goes higher at the peak and falls to zero somewhat slowly. Even if the temperature is high enough to randomize the spins *on the average*, there are still local regions where there is a certain amount of polarization, and in these regions the spins still have a little extra energy of interaction—which only dies out slowly as things get more and more random with further increases in temperature. So the actual curve looks like Fig. 37-2(c). One of the challenges of theoretical physics today is to find an exact theoretical description of the character of the specific heat near the Curie transition—an intriguing problem which has not yet been solved. Naturally, this problem is very closely related to the shape of the magnetization curve in the same region.

Now we want to describe some experiments, other than thermodynamic ones, which show that there is something *right* about our interpretation of magnetism. When the material is magnetized to saturation at low enough temperatures, M is very nearly equal to M_{sat} —nearly all the spins are parallel, as well as their magnetic moments. We can check this by an experiment. Suppose we suspend a bar magnet by a thin fiber and then surround it by a coil so that we can reverse the magnetic field without touching the magnet or putting any torque on it. This is a very difficult experiment because the magnetic forces are so enormous that any irregularities, any lopsidedness, or any lack of perfection in the iron will produce accidental torques. However, the experiment has been done under careful conditions in which such accidental torques are minimized. By means of the magnetic field from a coil that surrounds the bar, we turn all the atomic magnets over at once. When we do this we also change the angular momenta of all the spins from “up” to “down” (see Fig. 37-3). If angular momentum is to be conserved when the spins all turn over, the rest of the bar must have an opposite change in angular momentum. The whole magnet will start to spin. And sure enough, when we do the experiment, we find a slight turning of the magnet. We can measure the total angular momentum given to the whole magnet, and this is simply N times \hbar , the change in the angular momentum of each spin. The ratio of angular momentum to magnetic moment measured this way comes out to within about 10 percent of what we calculate. Actually, our calculations assume that the atomic magnets are due purely to the electron spin, but there is, in addition, some orbital motion also in most materials. The orbital motion is not completely free of the lattice and does not contribute much more than a few percent to the magnetism. As a matter of fact, the saturation magnetic field that one gets taking $M_{sat} = N\mu$ and using the density of iron of 7.9 and the moment μ of the spinning electron is about 20,000 gauss. But according to experiment, it is actually in the neighborhood of 21,500 gauss. This is a typical magnitude of error—5 or 10 percent—due to neglecting the contributions of the orbital moments that have not been included in making the analysis. Thus, a slight discrepancy with the gyromagnetic measurements is quite understandable.

37-3 The hysteresis curve

We have concluded from our theoretical analysis that a ferromagnetic material should spontaneously become magnetized below a certain temperature so that all the magnetism would be in the same direction. But we know that this is not true for an ordinary piece of *unmagnetized* iron. Why isn't all iron magnetized? We can explain it with the help of Fig. 37-4. Suppose the iron were all a big single crystal of the shape shown in Fig. 37-4(a) and spontaneously magnetized all in one direction. Then there would be a considerable external magnetic field, which would have a lot of energy. We can reduce that field energy if we arrange that one side of

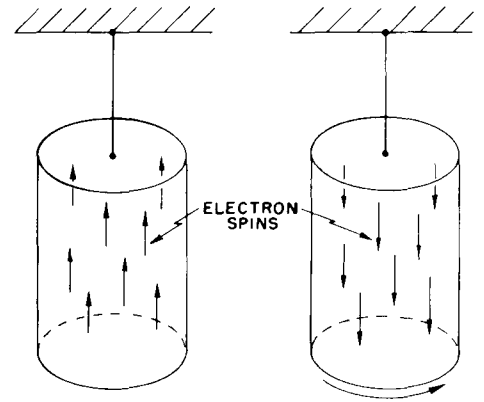


Fig. 37-3. When the magnetization of a bar of iron is reversed, the bar is given some angular velocity.

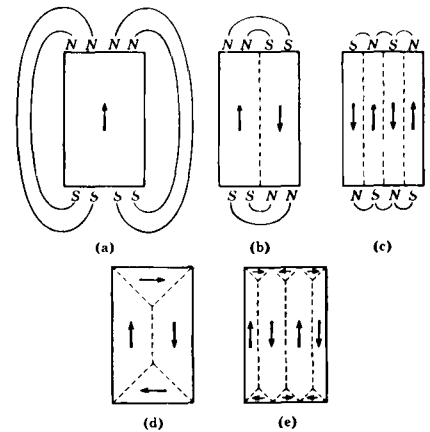


Fig. 37-4. The formation of domains in a single crystal of iron. [From Charles Kittel, *Introduction to Solid State Physics*, John Wiley and Sons, Inc., New York, 2nd ed., 1956.]

the block is magnetized “up” and the other side magnetized “down,” as in Fig. 37-4(b). Then, of course, the fields outside the iron would extend over less volume, so there would be less energy there.

Ah, but wait! In the layer between the two regions we have up-spinning electrons adjacent to down-spinning electrons. But ferromagnetism appears only in those materials for which the energy is *reduced* if the electrons are *parallel* rather than opposite. So, we have added some extra energy along the dotted line in Fig. 37-4(b); this energy is sometimes called *wall energy*. A region having only one direction of magnetization is called a *domain*. At the interface—the “wall”—between two domains, where we have atoms on opposite sides which are spinning in different directions, there is an energy per unit area of the wall. We have described it as though two adjacent atoms were spinning exactly opposite, but it turns out that nature adjusts things so that the transition is more gradual. But we don’t need to worry about such fine details at this point.

Now the question is: When is it better or worse to make a wall? The answer is that it depends on the *size* of the domains. Suppose that we were to scale up a block so that the whole thing was twice as big. The volume in the space outside filled with a given magnetic field strength would be *eight* times bigger, and the energy in the magnetic field, which is proportional to the volume, would also be eight times greater. But the *surface* area between two domains, which will give the wall energy, would be only *four* times as big. Therefore, if the piece of iron is big enough, it will pay to split it into more domains. This is why only the very tiny crystals can have but a single domain. Any large object—one more than about a hundredth of a millimeter in size—will have at least one domain wall; and any ordinary, “centimeter-size” object will be split into many domains, as shown in the figure. Splitting into domains goes on *until the energy needed to put in one extra wall is as large as the energy decrease in the magnetic field outside the crystal*.

Actually nature has discovered still another way to lower the energy: It is not necessary to have the field go outside at all, if a little triangular region is magnetized *sideways*, as in Fig. 37-4(d).^{*} Then with the arrangement of Fig. 37-4(d) we see that there is *no* external field, but instead only a little more domain wall.

But that introduces a new kind of problem. It turns out that when a single crystal of iron is magnetized, it changes its length in the direction of magnetization, so an “ideal” cube with its magnetization, say, “up,” is no longer a perfect cube. The “vertical” dimension will be different from the “horizontal” dimension. This effect is called *magnetostriction*. Because of such geometric changes, the little triangular pieces of Fig. 37-4(d) do not, so to speak, “fit” into the available space anymore—the crystal has got too long one way and too short the other way. Of course, it *does* fit, really, but only by being squashed in; and this involves some mechanical stresses. So, this arrangement *also* introduces an extra energy. It is the balance of all these various energies which determines how the domains finally arrange themselves in their complicated fashion in a piece of unmagnetized iron.

Now, what happens when we put on an external magnetic field? To take a simple case, consider a crystal whose domains are as shown in Fig. 37-4(d). If we apply an external magnetic field in the upward direction, in what manner does the crystal become magnetized? First, the middle domain wall can *move over sideways* (to the right) and reduce the energy. It moves over so that the region which is “up” becomes bigger than the region which is “down”. There are more elementary magnets lined up with the field, and this gives a lower energy. So, for a piece of iron in weak fields—at the very beginning of magnetization—the domain walls begin to move and eat into the regions which are magnetized opposite to the field. As the field continues to increase, a whole crystal shifts gradually into a single

^{*} You may be wondering how spins that have to be either “up” or “down” can also be “sideways”! That’s a good question, but we won’t worry about it right now. We’ll simply adopt the classical point of view, thinking of the atomic magnets as classical dipoles which can be polarized sideways. Quantum mechanics requires considerable expertness to understand how things can be quantized both “up-and-down,” and “right-and-left,” all at the same time.

large domain which the external field helps to keep lined up. In a strong field the crystal “likes” to be all one way *just because* its energy in the applied field is reduced—it is no longer merely the crystal’s own external field which matters.

What if the geometry is not so simple? What if the axes of the crystal and its spontaneous magnetization are in one direction, but we apply the magnetic field in *some other direction*—say at 45°? We might think that domains would reform themselves with their magnetization parallel to the field, and then as before, they could all grow into one domain. But this is not easy for the iron to do, *for the energy needed to magnetize a crystal depends on the direction of magnetization relative to the crystal axis*. It is relatively easy to magnetize iron in a direction parallel to the crystal axes, but it takes *more* energy to magnetize it in some other direction—like 45° with respect to one of the axes. Therefore, if we apply a magnetic field in such a direction, what happens first is that the domains which point along one of the preferred directions which is *near* to the applied field grow until the magnetization is all along one of these directions. Then *with much stronger fields*, the magnetization is gradually pulled around parallel to the field, as sketched in Fig. 37-5.

In Fig. 37-6 are shown some observations of the magnetization curves of single crystals of iron. To understand them, we must first explain something about the notation that is used in describing directions in a crystal. There are many ways in which a crystal can be sliced so as to produce a face which is a plane of atoms. Everyone who has driven past an orchard or vineyard knows this—it is fascinating to watch. If you look one way, you see lines of trees—if you look another way, you see different lines of trees, and so on. In a similar way, a crystal has definite families of planes that hold many atoms, and the planes have this important characteristic (we consider a cubic crystal to make it easier): If we observe where the planes intersect the three coordinate axes—we find that the *reciprocals* of the three distances from the origin are in the ratio of simple whole numbers. These three whole numbers are taken as the definition of the planes. For example, in Fig. 37-7(a), a plane parallel to the yz -plane is shown. This is called a [100] plane; the reciprocals of its intersection of the y - and z -axes are both zero. The direction perpendicular to such a plane (in a cubic crystal) is given the same set of numbers. It is easy to understand the idea in a cubic crystal, for then the indices [100] mean a vector which has a unit component in the x -direction and none in the y - or z -directions. The [110] direction is in a direction 45° from the x - and y -axes, as in Fig. 37-7(b); and the [111] direction is in the direction of the cube diagonal, as in Fig. 37-7(c).

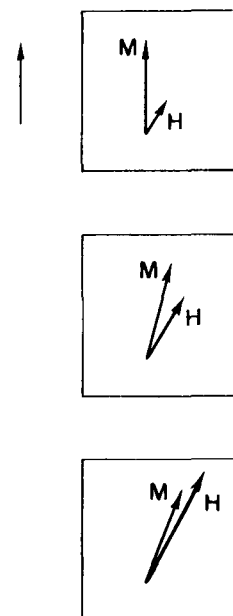


Fig. 37-5. A magnetizing field H at an angle with respect to the crystal axis will gradually change the direction of the magnetization without changing its magnitude.

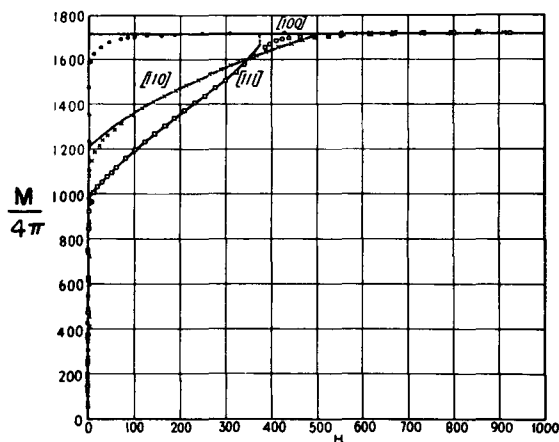


Fig. 37-6. The component of M parallel to H , for different directions of H (with respect to the crystal axes). [From F. Bitter, *Introduction to Ferromagnetism*, McGraw-Hill Book Co., Inc., 1937.]

Returning now to Fig. 37-6, we see the magnetization curves of a single crystal of iron for various directions. First, note that for very tiny fields—so weak that it is hard to see them on the scale at all—the magnetization increases extremely rapidly to quite large values. If the field is in the [100] direction—namely along one of those nice, easy directions of magnetization—the curve goes up to a high value, curves around a little, and then is saturated. What happened is that the

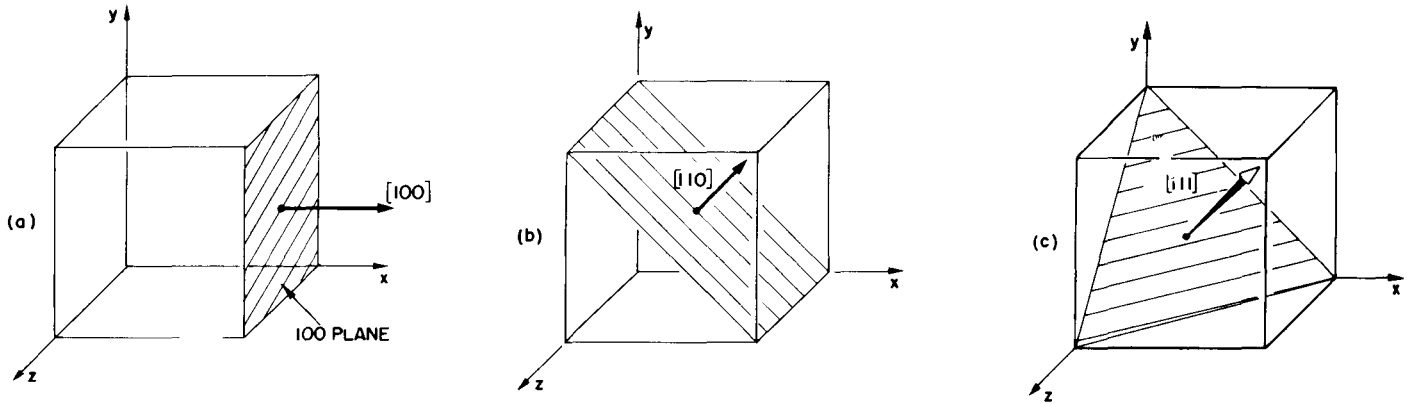


Fig. 37-7. The way the crystal planes are labeled.

domains which were already there are very easily removed. Only a small field is required to make the domain walls move and eat up all of the “wrong-way” domains. Single crystals of iron are enormously permeable (magnetic sense), much more so than ordinary polycrystalline iron. A perfect crystal magnetizes extremely easily. Why is it curved at all? Why doesn't it just go right up to saturation? We are not sure. You might study that some day. We do understand why it is flat for high fields. When the whole block is a single domain, the extra magnetic field cannot make any more magnetization—it is already at M_{sat} , with all the electrons lines up.

Now, if we try to do the same thing in the $[110]$ direction—which is at 45° to the crystal axes—what will happen? We turn on a little bit of field and the magnetization leaps up as the domains grow. Then as we increase the field some more, we find that it takes quite a lot of field to get up to saturation, because now the magnetization is *turning away* from an “easy” direction. If this explanation is correct, the point at which the $[110]$ curve extrapolates back to the vertical axis should be at $1/\sqrt{2}$ of the saturation value. It turns out, in fact, to be very, very close to $1/\sqrt{2}$. Similarly, in the $[111]$ direction—which is along the cube diagonal—we find, as we would expect, that the curve extrapolates back to nearly $1/\sqrt{3}$ of saturation.

Figure 37-8 shows the corresponding situation for two other materials, nickel and cobalt. Nickel is different from iron. In nickel, it turns out that the $[111]$ direction is the easy direction of magnetization. Cobalt has a hexagonal crystal form, and people have botched up the system of nomenclature for this case. They want to have three axes on the bottom of the hexagon and one perpendicular to these, so they have used four indices. The $[0001]$ direction is the direction of the axis of the hexagon, and $[10\bar{1}0]$ is perpendicular to that axis. We see that crystals of different metals behave in different ways.

Now we must discuss a polycrystalline material, such as an ordinary piece of iron. Inside such materials there are many, many little crystals with their crystalline axes pointing every which way. *These are not the same as domains.* Remember that the domains were all part of a *single crystal*, but in a piece of iron there are

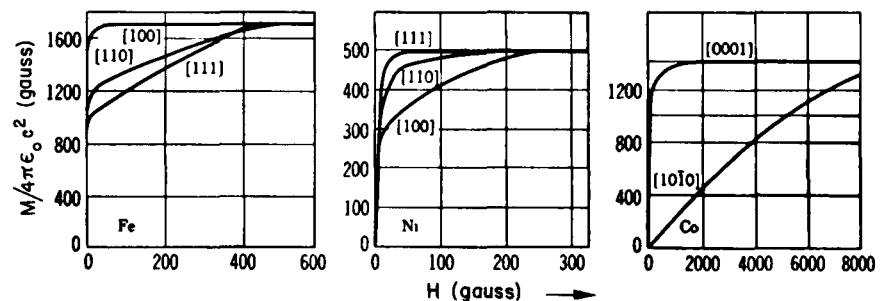


Fig. 37-8. Magnetization curves for single crystals of iron, nickel, and cobalt. [From Charles Kittel, *Introduction to Solid State Physics*, John Wiley and Sons, Inc., New York, 2nd ed., 1956.]

many *different crystals* with axes at different orientations, as shown in Fig. 37-9. Within each of these crystals, there will also generally be some domains. When we apply a *small* magnetic field to a piece of polycrystalline material, what happens is that the domain walls begin to move, and the domains which have a favorable direction of easy magnetization grow larger. This growth is reversible so long as the field stays very small—if we turn the field off, the magnetization will return to zero. This part of the magnetization curve is marked *a* in Fig. 37-10.

For larger fields—in the region *b* of the magnetization curve shown—things get much more complicated. In every small crystal of the material, there are strains and dislocations; there are impurities, dirt, and imperfections. And at all but the smallest fields, the domain wall, in moving, gets stuck on these. There is an interaction energy between the domain wall and a dislocation, or a grain boundary, or an impurity. So when the wall gets to one of them, it gets stuck; it sticks there at a certain field. But then if the field is raised some more, the wall suddenly snaps past. So the motion of the domain wall is not smooth the way it is in a perfect crystal—it gets hung up every once in a while and moves in jerks. If we were to look at the magnetization on a microscopic scale, we would see something like the insert of Fig. 37-10.

Now the important thing is that these jerks in the magnetization can cause an energy loss. In the first place, when a boundary finally slips past an impediment, it moves very quickly to the next one, since the field is already above what would be required for the unimpeded motion. The rapid motion means that there are rapidly changing magnetic fields which produce eddy currents in the crystal. These currents lose energy in heating the metal. A second effect is that when a domain suddenly changes, part of the crystal changes its dimensions from the magnetostriction. Each sudden shift of a domain wall sets up a little sound wave that carries away energy. Because of such effects, the second part of magnetization curve is *irreversible*, and *there is energy being lost*. This is the origin of the hysteresis effect, because to move a boundary wall forward—snap—and then to move it backward—snap—produces a different result. It's like "jerky" friction, and it takes energy.

Eventually, for high enough fields, when we have moved all the domain walls and magnetized each crystal in its best direction, there are still some crystallites which happen to have their easy directions of magnetization not in the direction of our external magnetic field. Then it takes a lot of extra field to turn those magnetic moments around. So the magnetization increases slowly, but smoothly, for high fields—namely in the region marked *c* in the figure. The magnetization does not come sharply to its saturation value, because in the last part of the curve the atomic magnets are *turning* in the strong field. So we see why the magnetization curve of an ordinary polycrystalline materials, such as the one shown in Fig. 37-10, rises a little bit and *reversibly* at first, then rises irreversibly, and then curves over slowly. Of course, there is no sharp break-point between the three regions—they blend smoothly, one into the other.

It is not hard to show that the magnetization process in the middle part of the magnetization curve is jerky—that the domain walls jerk and snap as they shift. All you need is a coil of wire—with many thousands of turns—connected to an amplifier and a loudspeaker, as shown in Fig. 37-11. If you put a few silicon steel sheets (of the type used in transformers) at the center of the coil and bring a bar magnet slowly near the stack, the sudden changes in magnetization will produce impulses of emf in the coil, which are heard as distinct clicks in the loudspeaker. As you move the magnet nearer to the iron you will hear a whole rush of clicks that sound something like the noise of sand grains falling over each other as a can of sand is tilted. The domain walls are jumping, snapping, and jiggling as the field is increased. This phenomenon is called the *Barkhausen effect*.

As you move the magnet even closer to the iron sheets, the noise grows louder and louder for a while but then there is relatively little noise when the magnet gets very close. Why? Because nearly all the domain walls have moved as far as they can go. Any greater field is merely *turning* the magnetization in each domain, which is a smooth process.

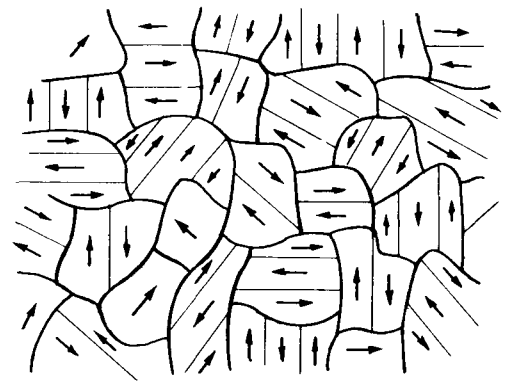


Fig. 37-9. The microscopic structure of an unmagnetized ferromagnetic material. Each crystal grain has an easy direction of magnetization and is broken up into domains which are spontaneously magnetized (usually) parallel to this direction.

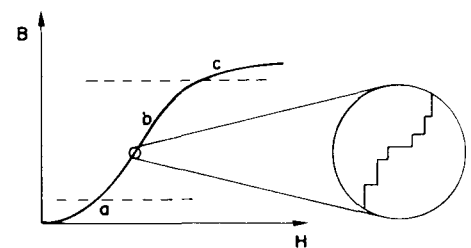


Fig. 37-10. The magnetization curve for polycrystalline iron.

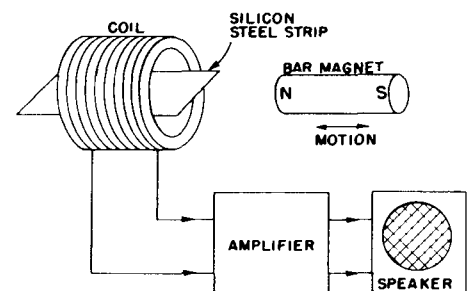


Fig. 37-11. The sudden changes in the magnetization of the steel strip are heard as clicks in the loudspeaker.

If you now withdraw the magnet, so as to come back on the downward branch of the hysteresis loop, the domains all try to get back to low energy again, and you hear another rush of backward-going jerks. You can also note that if you bring the magnet to a given place and move it back and forth a little bit, there is relatively little noise. It is again like tilting a can of sand—once the grains shift into place, small movements of the can don't disturb them. In the iron the small variations in the magnetic field aren't enough to move any boundaries over any of the "humps."

37-4 Ferromagnetic materials

Now we would like to talk about the various kinds of magnetic materials that there are in the technical world and to consider some of the problems involved in designing magnetic materials for different purposes. First, the term "the magnetic properties of iron," which one often hears, is a misnomer—there is no such thing. "Iron" is not a well-defined material—the properties of iron depend critically on the amount of impurities and also on *how* the iron is formed. You can appreciate that the magnetic properties will depend on how easily the domain walls move and that this is a *gross* property, not a property of the individual atoms. So practical ferromagnetism is not really a property of an iron *atom*—it is a property of *solid iron* in a *certain form*. For example, iron can take on two different crystalline forms. The common form has a body-centered cubic lattice, but it can also have a face-centered cubic lattice, which is, however, stable only at temperatures above 1100°C. Of course, at that temperature the body-centered cubic structure is already past the Curie point. However, by alloying chromium and nickel with the iron (one possible mixture is 18 percent chromium and 8 percent nickel) we can get what is called stainless steel, which, although it is mainly iron, retains the face-centered lattice even at low temperatures. Because its crystal structure is different, it has completely different magnetic properties. Most kinds of stainless steel are not magnetic to any appreciable degree, although there are some kinds which are somewhat magnetic—it depends on the composition of the alloy. Even when such an alloy is magnetic, it is not *ferromagnetic* like ordinary iron—even though it is mostly just iron.

We would like now to describe a few of the special materials which have been developed for their particular magnetic properties. First, if we want to make a *permanent* magnet, we would like material with an enormously *wide* hysteresis loop so that, when we turn the current off and come down to zero magnetizing field, the magnetization will remain large. For such materials the domain boundaries should be "frozen" in place as much as possible. One such material is the remarkable alloy "Alnico V" (51% Fe, 8% Al, 14% Ni, 24% Co, 3% Cu). (The rather complex composition of this alloy is indicative of the kind of detailed effort that has gone into making good magnets. What patience it takes to mix five things together and test them until you find the most ideal substance!) When Alnico solidifies, there is a "second phase" which precipitates out, making many tiny grains and very high internal strains. In this material, the domain boundaries have a hard time moving at all. In addition to having a precise composition, Alnico is mechanically "worked" in a way that makes the crystals appear in the form of long grains along the direction in which the magnetization is going to be. Then the magnetization will have a natural tendency to be lined up in these directions and will be held there from the anisotropic effects. Furthermore, the material is even cooled in an external magnetic field when it is manufactured, so that the grains will grow with the right crystal orientation. The hysteresis loop of Alnico V is shown in Fig 37-12. You see that it is about 500 times wider than the hysteresis curve for soft iron that we showed in the last chapter in Fig. 36-8.

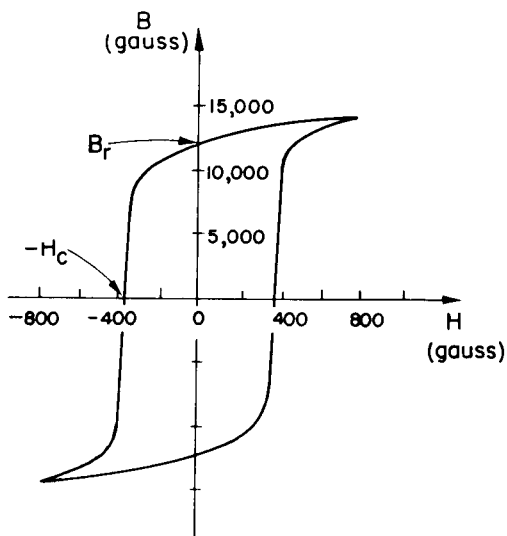


Fig 37-12. The hysteresis curve of Alnico V.

Let's turn now to a different kind of material. For building transformers and motors, we want a material which is magnetically "soft"—one in which the magnetism is easily changed so that an enormous amount of magnetization results from a very small applied field. To arrange this, we need pure, well-annealed material which will have very few dislocations and impurities so that the domain

walls can move easily. It would also be nice if we could make the anisotropy small. Then, even if a grain of the material sits at the wrong angle with respect to the field, it will still magnetize easily. Now we have said that iron prefers to magnetize along the [100] direction, whereas nickel prefers the [111] direction; so if we mix iron and nickel in various proportions, we might hope to find that with just the right proportions the alloy wouldn't prefer *any* direction—the [100] and [111] directions would be equivalent. It turns out that this happens with a mixture of 70 percent nickel and 30 percent iron. In addition—possibly by luck or maybe because of some physical relationship between the anisotropy and the magnetostriction effects—it turns out that the *magnetostriction* of iron and nickel has the opposite sign. And in an alloy of the two metals, this property goes through zero at about 80 percent nickel. So somewhere between 70 and 80 percent nickel we get very “soft” magnetic materials—alloys that are very easy to magnetize. They are called the *permalloys*. Permalloys are useful for high-quality transformers (at low signal levels), but they would be no good at all for permanent magnets. Permalloys must be very carefully made and handled. The magnetic properties of a piece of permalloy are drastically changed if it is stressed beyond its elastic limit—it mustn't be bent. Then, its permeability is reduced because of the dislocations, slip bands, and so on, which are produced by the mechanical deformations. The domain boundaries are no longer easy to move. The high permeability can, however, be restored by annealing at high temperatures.

It is often convenient to have some numbers to characterize the various magnetic materials. Two useful numbers are the intercepts of the hysteresis loop with the B - and H -axes, as indicated in Fig. 37-12. These intercepts are called the *remanent magnetic field* B_r and the *coercive force* H_c . In Table 37-1 we list these numbers for a few magnetic materials.

Table 37-1

Properties of some ferromagnetic materials

Material	B_r Residual magnetic field (gauss)	H_c Coercive force (gauss)
Supermalloy	(≈ 5000)	0.004
Silicon steel (transformer)	12,000	0.05
Armco iron	4000	0.6
Alnico V	13,000	550.

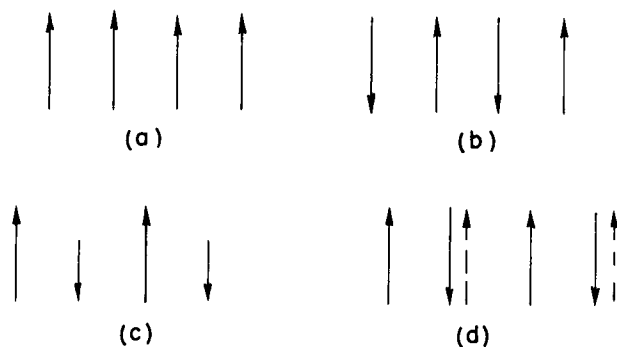


Fig. 37-13. Relative orientation of electron spins in various materials: (a) ferromagnetic, (b) antiferromagnetic, (c) ferrite, (d) yttrium-iron alloy. (Broken arrows show direction of total angular momentum, including orbital motion.)

37-5 Extraordinary magnetic materials

We would now like to discuss some of the more exotic magnetic materials. There are many elements in the periodic table which have incomplete inner electron shells and hence have atomic magnetic moments. For instance, right next to the ferromagnetic elements iron, nickel, and cobalt you will find chromium and manganese. Why aren't *they* ferromagnetic? The answer is that the λ term in Eq. (37.1) has the *opposite sign* for these elements. In the chromium lattice, for example, the spins of the chromium atoms alternate *atom by atom*, as shown in Fig. 37-13(b). So chromium *is* “magnetic” from its own point of view, but it is not technically interesting because there are no *external* magnetic effects. Chromium, then, is an example of a material in which quantum mechanical effects make the spins alternate. Such a material is called *antiferromagnetic*. The alignment in antiferromagnetic materials is also temperature dependent. Below a critical temperature, all the spins are lined up in the alternating array, but when the material is heated above a certain temperature—which is again called the Curie temperature—the spins suddenly become random. There is, internally, a sudden transition. This transition can be seen in the specific heat curve. Also it shows up in some special “magnetic” effects. For instance, the existence of the alternating spins can be verified by scattering neutrons from a crystal of chromium. Because a neutron itself has a spin

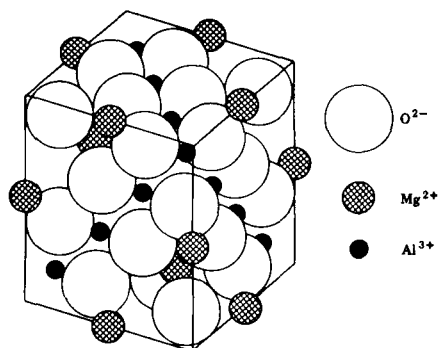


Fig. 37-14. Crystal structure of the mineral spinel (MgAl_2O_4); the Mg^{+2} ions occupy tetrahedral sites, each surrounded by four oxygen ions; the Al^{+3} ions occupy octahedral sites, each surrounded by six oxygen ions. [From Charles Kittel, *Introduction to Solid State Physics*, John Wiley and Sons, Inc., New York, 2nd ed., 1956]

(and a magnetic moment), it has a different amplitude to be scattered, depending on whether its spin is parallel or opposite to the spin of the scatterer. Thus, we get a different interference pattern when the spins in a crystal are alternating than we do when they have a random distribution

There is another kind of substance in which quantum mechanical effects make the electron spins alternate, but which is nevertheless *ferromagnetic*—that is, the crystal has a permanent net magnetization. The idea behind such materials is shown in Fig. 37-14. The figure shows the crystal structure of *spinel*, a magnesium-aluminum oxide, which—as it is shown—is *not* magnetic. The oxide has two kinds of metal atoms: magnesium and aluminum. Now if we replace the magnesium and the aluminum by two magnetic elements like iron and zinc, or by zinc and manganese—in other words, if we put in *magnetic* atoms instead of the nonmagnetic ones—an interesting thing happens. Let's call one kind of metal atom *a* and the other kind of metal atom *b*; then the following combination of forces must be considered. There is an *a-b* interaction which tries to make the *a* atoms and the *b* atoms have opposite spins—because quantum mechanics always gives the opposite sign (except for the mysterious crystals of iron, nickel, and cobalt). Then, there is a direct *a-a* interaction which tries to make the *a*'s opposite, and also a *b-b* interaction which tries to make the *b*'s opposite. Now, of course we cannot have everything opposite everything else—*a* opposite *b*, *a* opposite *a*, and *b* opposite *b*. Presumably because of the distances between the *a*'s and the presence of the oxygen (although we really don't know why), it turns out that the *a-b* interaction is stronger than the *a-a* or the *b-b*. So the solution that nature uses in this case is to make all the *a*'s *parallel to each other*, and all the *b*'s *parallel to each other*, but the two systems *opposite*. That gives the lowest energy because of the stronger *a-b* interaction. The result: all the *a*'s are spinning up and all the *b*'s are spinning down—or vice versa, of course. But if the *magnetic moments* of the *a*-type atom and the *b*-type atom *are not equal*, we can get the situation shown in Fig. 37-13(c), and there can be a net magnetization in the material. The material will then be ferromagnetic—although somewhat weak. Such materials are called *ferrites*. They do not have as high a saturation magnetization as iron—for obvious reasons—so they are only useful for smaller fields. But they have a very important difference—they are insulators; the ferrites are *ferromagnetic insulators*. In high-frequency fields, they will have very small eddy currents and so can be used, for example, in microwave systems. The microwave fields will be able to get inside such an insulating material, whereas they would be kept out by the eddy currents in a conductor like iron.

There is another class of magnetic materials which has only recently been discovered—members of the family of the orthosilicates called *garnets*. They are again crystals in which the lattice contains two kinds of metallic atoms, and we have again a situation in which two kinds of atoms can be substituted almost at will. Among the many compounds of interest there is one which is completely ferromagnetic. It has yttrium and iron in the garnet structure, and the reason it is ferromagnetic is very curious. Here again quantum mechanics is making the neighboring spins opposite, so that there is a locked-in system of spins with the electron spins of the iron one way and the electron spins of the yttrium the opposite way. But the yttrium atom is complicated. It is a rare-earth element and gets a large contribution to its magnetic moment from *orbital* motion of the electrons. For yttrium, the orbital motion contribution is *opposite* that of the spin and also is bigger. Thus, although quantum mechanics, working through the exclusion principle, makes the *spins* of the yttrium opposite those of the iron, it makes the *total* magnetic moment of the yttrium atom *parallel* to the iron because of the orbital effect—as sketched in Fig. 37-13(d). The compound is therefore a regular ferromagnet.

Another interesting example of ferromagnetism occurs in some of the rare-earth elements. It has to do with a still more peculiar arrangement of the spins. The material is not ferromagnetic in the sense that the spins are all parallel, nor is it antiferromagnetic in the sense that every atom is opposite. In these crystals all of the spins *in one layer* are parallel and lie in the plane of the layer. In the next

layer all spins are again parallel to each other, but point in a somewhat different direction. In the following layer they are in still another direction, and so on. The result is that the local magnetization vector varies in the form of a spiral—the magnetic moments of the successive layers rotate as we proceed along a line perpendicular to the layers. It is interesting to try to analyze what happens when a field is applied to such a spiral—all the twistings and turnings that must go on in all those atomic magnets. (Some people *like* to amuse themselves with the theory of these things!) Not only are there cases of “flat” spirals, but there are also cases in which the directions the magnetic moments of successive layers map out a cone, so that it has a spiral component and also a uniform ferromagnetic component in one direction!

The magnetic properties of materials, worked out on a more advanced level than we have been able to do here, have fascinated physicists of all kinds. In the first place, there are those practical people who love to work out ways of making things in a better way—they love to design better and more interesting magnetic materials. The discovery of things like ferrites, or their application, immediately delights people who like to see clever new ways of doing things. Besides this, there are those who find a fascination in the terrible complexity that nature can produce using a few basic laws. Starting with one and the same general idea, nature goes from the ferromagnetism of iron and its domains, to the antiferromagnetism of chromium, to the magnetism of ferrites and garnets, to the spiral structure of the rare earth elements, and on, and on. It is fascinating to discover experimentally all the strange things that go on in these special substances. Then, to the theoretical physicists, ferromagnetism presents a number of very interesting, unsolved, and beautiful challenges. One challenge is to understand why it exists at all. Another is to predict the statistics of the interacting spins in an ideal lattice. Even neglecting any possible extraneous complications, this problem has, so far, defied full understanding. The reason that it is so interesting is that it is such an easily stated problem: Given a lot of electron spins in a regular lattice, interacting with such-and-such a law, what do they do? It is simply stated, but it has defied complete analysis for years. Although it has been analyzed rather carefully for temperatures not too close to the Curie point, the theory of the sudden transition ~~at~~ the Curie point still needs to be completed.

Finally, the whole subject of the system of spinning atomic magnets—in ferromagnetic, or in paramagnetic materials and in nuclear magnetism, has also been a fascinating thing to advanced students in physics. The system of spins can be pushed on and pulled on with external magnetic fields, so one can do many tricks with resonances, with relaxation effects, with spin-echoes, and with other effects. It serves as a prototype of many complicated thermodynamic systems. But in paramagnetic materials the situation is often fairly simple, and people have been delighted both to do experiments and to explain the phenomena theoretically.

We now close our study of electricity and magnetism. In the first chapter, we spoke of the great strides that have been made since the early Greek observation of the strange behaviors of amber and of lodestone. Yet in all our long and involved discussion we have never explained *why it is that when we rub a piece of amber we get a charge on it*, nor have we explained *why a lodestone is magnetized!* You may say, “Oh, we just didn’t get the right sign.” No, it is worse than that. Even if we *did* get the right sign, we would still have the question: Why is the piece of lodestone in the ground magnetized? There is the earth’s magnetic field, of course, but *where does the earth’s field come from?* Nobody really knows—there have only been some good guesses. So you see, this physics of ours is a lot of fakery—we start out with the phenomena of lodestone and amber, and we end up not understanding either of them very well. But we *have* learned a tremendous amount of very exciting and very practical information in the process!