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Electromagnetism

1-1 Electrical forces

Consider a force like gravitation which varies predominantly inversely as the square of the distance, but which is about a *billion-billion-billion-billion* times stronger. And with another difference. There are two kinds of "matter," which we can call positive and negative. Like kinds repel and unlike kinds attract— unlike gravity where there is only attraction. What would happen?

A bunch of positives would repel with an enormous force and spread out in all directions. A bunch of negatives would do the same. But an evenly mixed bunch of positives and negatives would do something completely different. The opposite pieces would be pulled together by the enormous attractions. The net result would be that the terrific forces would balance themselves out almost perfectly, by forming tight, fine mixtures of the positive and the negative, and between two separate bunches of such mixtures there would be practically no attraction or repulsion at all.

There is such a force: the electrical force. And all matter is a mixture of positive protons and negative electrons which are attracting and repelling with this great force. So perfect is the balance, however, that when you stand near someone else you don't feel any force at all. If there were even a little bit of unbalance you would know it. If you were standing at arm's length from someone and each of you had *one percent* more electrons than protons, the repelling force would be incredible. How great? Enough to lift the Empire State Building? No! To lift Mount Everest? No! The repulsion would be enough to lift a "weight" equal to that of the entire earth!

With such enormous forces so perfectly balanced in this intimate mixture, it is not hard to understand that matter, trying to keep its positive and negative charges in the finest balance, can have a great stiffness and strength. The Empire State Building, for example, swings only eight feet in the wind because the electrical forces hold every electron and proton more or less in its proper place. On the other hand, if we look at matter on a scale small enough that we see only a few atoms, any small piece will not, usually, have an equal number of positive and negative charges, and so there will be strong residual electrical forces. Even when there are equal numbers of both charges in two neighboring small pieces, there may still be large net electrical forces because the forces between individual charges vary inversely as the square of the distance. A net force can arise if a negative charge of one piece is closer to the positive than to the negative charges of the other piece. The attractive forces can then be larger than the repulsive ones and there can be a net attraction between two small pieces with no excess charges. The force that holds the atoms together, and the chemical forces that hold molecules together, are really electrical forces acting in regions where the balance of charge is not perfect, or where the distances are very small.

You know, of course, that atoms are made with positive protons in the nucleus and with electrons outside. You may ask: "If this electrical force is so terrific, why don't the protons and electrons just get on top of each other? If they want to be in an intimate mixture, why isn't it still more intimate?" The answer has to do with the quantum effects. If we try to confine our electrons in a region that is very close to the protons, then according to the uncertainty principle they must have some mean square momentum which is larger the more we try to confine them. It is this motion, required by the laws of quantum mechanics, that keeps the electrical attraction from bringing the charges any closer together.

There is another question: "What holds the nucleus together"? In a nucleus there are several protons, all of which are positive. Why don't they push themselves apart? It turns out that in nuclei there are, in addition to electrical forces, nonelectrical forces, called nuclear forces, which are greater than the electrical forces and which are able to hold the protons together in spite of the electrical repulsion. The nuclear forces, however, have a short range—their force falls off much more rapidly than $1/r^2$. And this has an important consequence. If a nucleus has too many protons in it, it gets too big, and it will not stay together. An example is uranium, with 92 protons. The nuclear forces act mainly between each proton (or neutron) and its nearest neighbor, while the electrical forces act over larger distances, giving a repulsion between each proton and all of the others in the nucleus. The more protons in a nucleus, the stronger is the electrical repulsion, until, as in the case of uranium, the balance is so delicate that the nucleus is almost ready to fly apart from the repulsive electrical force. If such a nucleus is just "tapped" lightly (as can be done by sending in a slow neutron), it breaks into two pieces, each with positive charge, and these pieces fly apart by electrical repulsion. The energy which is liberated is the energy of the atomic bomb. This energy is usually called "nuclear" energy, but it is really "electrical" energy released when electrical forces have overcome the attractive nuclear forces.

We may ask, finally, what holds a negatively charged electron together (since it has no nuclear forces). If an electron is all made of one kind of substance, each part should repel the other parts. Why, then, doesn't it fly apart? But does the electron have "parts"? Perhaps we should say that the electron is just a point and that electrical forces only act between *different* point charges, so that the electron does not act upon itself. Perhaps. All we can say is that the question of what holds the electron together has produced many difficulties in the attempts to form a complete theory of electromagnetism. The question has never been answered. We will entertain overselves by discussing this subject some more in later chapters.

As we have seen, we should expect that it is a combination of electrical forces and quantum-mechanical effects that will determine the detailed structure of materials in bulk, and, therefore, their properties. Some materials are hard, some are soft. Some are electrical "conductors"—because their electrons are free to move about; others are "insulators"—because their electrons are held tightly to individual atoms. We shall consider later how some of these properties come about, but that is a very complicated subject, so we will begin by looking at the electrical forces only in simple situations. We begin by treating only the laws of electricity—including magnetism, which is really a part of the same subject.

We have said that the electrical force, like a gravitational force, decreases inversely as the square of the distance between charges. This relationship is called Coulomb's law. But it is not precisely true when charges are moving—the electrical forces depend also on the motions of the charges in a complicated way. One part of the force between moving charges we call the *magnetic* force. It is really one aspect of an electrical effect. That is why we call the subject "electromagnetism."

There is an important general principle that makes it possible to treat electromagnetic forces in a relatively simple way. We find, from experiment, that the force that acts on a particular charge—no matter how many other charges there are or how they are moving—depends only on the position of that particular charge, on the velocity of the charge, and on the amount of charge. We can write the force F on a charge q moving with a velocity v as

$$F = q(E + v \times B). \tag{1.1}$$

We call E the *electric field* and B the *magnetic field* at the location of the charge. The important thing is that the electrical forces from all the other charges in the universe can be summarized by giving just these two vectors. Their values will depend on *where* the charge is, and may change with *time*. Furthermore, if we replace that charge with another charge, the force on the new charge will be just in proportion to the amount of charge so long as all the rest of the charges in the

		Lower case Greek letters and commonly used capitals	
α		alpha
β		beta
γ	Γ	gamma
δ	Δ	delta
ϵ		epsilon
ζ		zeta
η		eta
θ	Θ	theta
ι		iota
κ		kappa
λ	Λ	lambda
μ		mu
ν		nu
ξ	Ξ	xi (ksi)
o		omicron
π	Π	pi
ρ		rho
σ	Σ	sigma
τ		tau
υ	Υ	upsilon
ϕ	Φ	phi
χ		chi (khi)
ψ	Ψ	psi
ω	Ω	omega

world do not change their positions or motions. (In real situations, of course, each charge produces forces on all other charges in the neighborhood and may cause these other charges to move, and so in some cases the fields *can* change if we replace our particular charge by another.)

We know from Vol. I how to find the motion of a particle if we know the force on it. Equation (1.1) can be combined with the equation of motion to give

$$\frac{d}{dt}\left[\frac{mv}{(1 - v^2/c^2)^{1/2}}\right] = F = q(E + v \times B). \tag{1.2}$$

So if E and B are given, we can find the motions. Now we need to know how the E's and B's are produced.

One of the most important simplifying principles about the way the fields are produced is this: Suppose a number of charges moving in some manner would produce a field E_1, and another set of charges would produce E_2. If both sets of charges are in place at the same time (keeping the same locations and motions they had when considered separately), then the field produced is just the sum

$$E = E_1 + E_2. \tag{1.3}$$

This fact is called *the principle of superposition* of fields. It holds also for magnetic fields.

This principle means that if we know the law for the electric and magnetic fields produced by a *single* charge moving in an arbitrary way, then all the laws of electrodynamics are complete. If we want to know the force on charge A we need only calculate the E and B produced by each of the charges B, C, D, etc., and then add the E's and B's from all the charges to find the fields, and from them the forces acting on charge A. If it had only turned out that the field produced by a single charge was simple, this would be the neatest way to describe the laws of electrodynamics. We have already given a description of this law (Chapter 28, Vol. I) and it is, unfortunately, rather complicated.

It turns out that the form in which the laws of electrodynamics are simplest are not what you might expect. It is *not* simplest to give a formula for the force that one charge produces on another. It is true that when charges are standing still the Coulomb force law is simple, but when charges are moving about the relations are complicated by delays in time and by the effects of acceleration, among others. As a result, we do not wish to present electrodynamics only through the force laws between charges; we find it more convenient to consider another point of view—a point of view in which the laws of electrodynamics appear to be the most easily manageable.

1-2 Electric and magnetic fields

First, we must extend, somewhat, our ideas of the electric and magnetic vectors, E and B. We have defined them in terms of the forces that are felt by a charge. We wish now to speak of electric and magnetic fields *at a point* even when there is no charge present. We are saying, in effect, that since there are forces "acting on" the charge, there is still "something" there when the charge is removed. If a charge located at the point (x, y, z) at the time t feels the force F given by Eq. (1.1) we associate the vectors E and B with *the point* in space (x, y, z). We may think of $E(x, y, z, t)$ and $B(x, y, z, t)$ as giving the forces that *would be* experienced at the time t by a charge located at (x, y, z), *with the condition* that placing the charge there *did not disturb* the positions or motions of all the other charges responsible for the fields.

Following this idea, we associate with *every* point (x, y, z) in space two vectors E and B, which may be changing with time. The electric and magnetic fields are, then, viewed as *vector functions* of x, y, z, and t. Since a vector is specified by its components, each of the fields $E(x, y, z, t)$ and $B(x, y, z, t)$ represent three mathematical functions of x, y, z, and t.
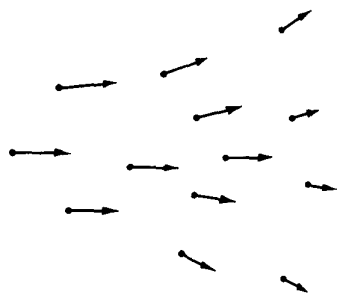
Fig. 1-1. A vector field may be represented by drawing a set of arrows whose magnitudes and directions indicate the values of the vector field at the points from which the arrows are drawn.
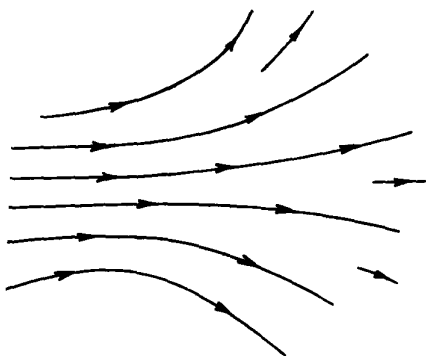
Fig. 1-2. A vector field can be represented by drawing lines which are tangent to the direction of the field vector at each point, and by drawing the density of lines proportional to the magnitude of the field vector.
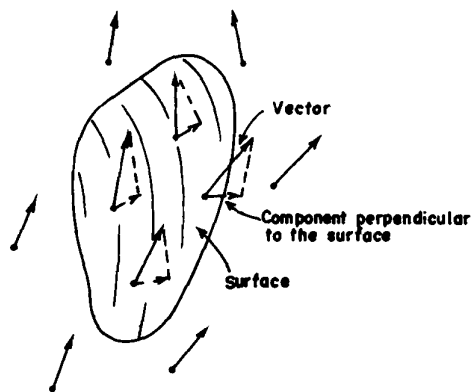
Fig. 1-3. The flux of a vector field through a surface is defined as the average value of the normal component of the vector times the area of the surface.

It is precisely because E (or B) can be specified at every point in space that it is called a "field." A "field" is any physical quantity which takes on different values at different points in space. Temperature, for example, is a field—in this case a scalar field, which we write as $T(x, y, z)$. The temperature could also vary in time, and we would say the temperature field is time-dependent, and write $T(x, y, z, t)$. Another example is the "velocity field" of a flowing liquid. We write $v(x, y, z, t)$ for the velocity of the liquid at each point in space at the time t. It is a vector field.

Returning to the electromagnetic fields—although they are produced by charges according to complicated formulas, they have the following important characteristic: the relationships between the values of the fields at *one point* and the values at a *nearby point* are very simple. With only a few such relationships in the form of differential equations we can describe the fields completely. It is in terms of such equations that the laws of electrodynamics are most simply written.

There have been various inventions to help the mind visualize the behavior of fields. The most correct is also the most abstract: we simply consider the fields as mathematical functions of position and time. We can also attempt to get a mental picture of the field by drawing vectors at many points in space, each of which gives the field strength and direction at that point. Such a representation is shown in Fig. 1-1. We can go further, however, and draw lines which are everywhere tangent to the vectors—which, so to speak, follow the arrows and keep track of the direction of the field. When we do this we lose track of the *lengths* of the vectors, but we can keep track of the strength of the field by drawing the lines far apart when the field is weak and close together when it is strong. We adopt the convention that the *number of lines per unit area* at right angles to the lines is proportional to the *field strength*. This is, of course, only an approximation, and it will require, in general, that new lines sometimes start up in order to keep the number up to the strength of the field. The field of Fig. 1-1 is represented by field lines in Fig. 1-2.

1-3 Characteristics of vector fields

There are two mathematically important properties of a vector field which we will use in our description of the laws of electricity from the field point of view. Suppose we imagine a closed surface of some kind and ask whether we are losing "something" from the inside; that is, does the field have a quality of "outflow"? For instance, for a velocity field we might ask whether the velocity is always outward on the surface or, more generally, whether more fluid flows out (per unit time) than comes in. We call the net amount of fluid going out through the surface per unit time the "flux of velocity" through the surface. The flow through an element of a surface is just equal to the component of the velocity perpendicular to the surface times the area of the surface. For an arbitrary closed surface, the *net outward flow*—or *flux*—is the average outward normal component of the velocity, times the area of the surface:

$$\text{Flux} = (\text{average normal component}) \cdot (\text{surface area}). \qquad (1.4)$$

In the case of an electric field, we can mathematically define something analogous to an outflow, and we again call it the flux, but of course it is not the flow of any substance, because the electric field is not the velocity of anything. It turns out, however, that the mathematical quantity which is the average normal component of the field still has a useful significance. We speak, then, of the *electric flux*—also defined by Eq. (1.4). Finally, it is also useful to speak of the flux not only through a completely closed surface, but through any bounded surface. As before, the flux through such a surface is defined as the average normal component of a vector times the area of the surface. These ideas are illustrated in Fig. 1-3.

There is a second property of a vector field that has to do with a line, rather than a surface. Suppose again that we think of a velocity field that describes the flow of a liquid. We might ask this interesting question: Is the liquid circulating?

By that we mean: Is there a net rotational motion around some loop? Suppose that we instantaneously freeze the liquid everywhere except inside of a tube which is of uniform bore, and which goes in a loop that closes back on itself as in Fig. 1–4. Outside of the tube the liquid stops moving, but inside the tube it may keep on moving because of the momentum in the trapped liquid—that is, if there is more momentum heading one way around the tube than the other. We define a quantity called the *circulation* as the resulting speed of the liquid in the tube times its circumference. We can again extend our ideas and define the "circulation" for any vector field (even when there isn't anything moving). For any vector field the *circulation around any imagined closed curve* is defined as the average tangential component of the vector (in a consistent sense) multiplied by the circumference of the loop (Fig. 1–5).

$$\text{Circulation} = (\text{average tangential component}){\cdot}(\text{distance around}). \quad (1.5)$$

You will see that this definition does indeed give a number which is proportional to the circulation velocity in the quickly frozen tube described above.

With just these two ideas—flux and circulation—we can describe all the laws of electricity and magnetism at once. You may not understand the significance of the laws right away, but they will give you some idea of the way the physics of electromagnetism will be ultimately described.

1–4 The laws of electromagnetism

The first law of electromagnetism describes the flux of the electric field:

$$\text{The flux of } E \text{ through any closed surface} = \frac{\text{the net charge inside}}{\epsilon_0}, \quad (1.6)$$

where ϵ_0 is a convenient constant. (The constant ϵ_0 is usually read as "epsilon-zero" or "epsilon-naught".) If there are no charges inside the surface, even though there are charges nearby outside the surface, the *average* normal component of E is zero, so there is no net flux through the surface. To show the power of this type of statement, we can show that Eq. (1.6) is the same as Coulomb's law, provided only that we also add the idea that the field from a single charge is spherically symmetric. For a point charge, we draw a sphere around the charge. Then the average normal component is just the value of the magnitude of E at any point, since the field must be directed radially and have the same strength for all points on the sphere. Our rule now says that the field at the surface of the sphere, times the area of the sphere—that is, the outgoing flux—is proportional to the charge inside. If we were to make the radius of the sphere bigger, the area would increase as the square of the radius. The average normal component of the electric field times that area must still be equal to the same charge inside, and so the field must decrease as the square of the distance—we get an "inverse square" field.

If we have an arbitrary curve in space and measure the circulation of the electric field around the curve, we will find that it is not, in general, zero (although it is for the Coulomb field). Rather, for electricity there is a second law that states: for any surface S (not closed) whose edge is the curve C,

$$\text{Circulation of } E \text{ around } C = \frac{d}{dt} (\text{flux of } B \text{ through } S). \quad (1.7)$$

We can complete the laws of the electromagnetic field by writing two corresponding equations for the magnetic field B.

$$\text{Flux of } B \text{ through any closed surface} = 0. \quad (1.8)$$

For a surface S bounded by the curve C,

$$c^2(\text{circulation of } B \text{ around } C) = \frac{d}{dt} (\text{flux of } E \text{ through } S)$$
$$+ \frac{\text{flux of electric current through } S}{\epsilon_0}. \quad (1.9)$$
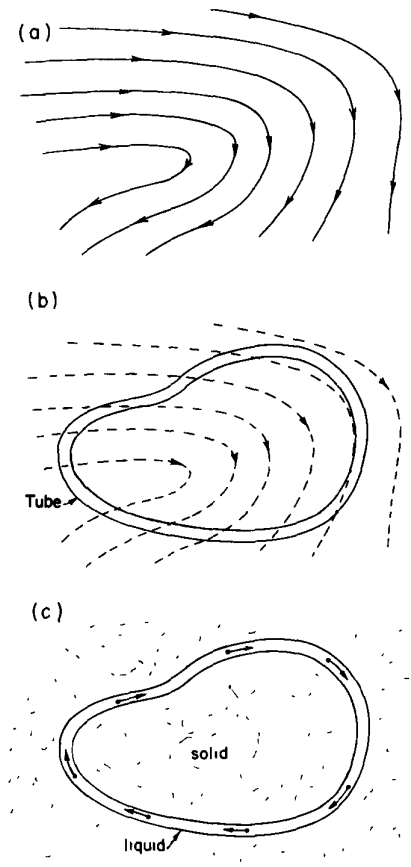
Fig. 1–4. (a) The velocity field in a liquid. Imagine a tube of uniform cross section that follows an arbitrary closed curve as in (b). If the liquid were suddenly frozen everywhere except inside the tube, the liquid in the tube would circulate as shown in (c).
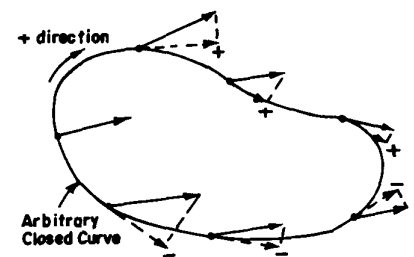
Fig. 1–5. The circulation of a vector field is the average tangential component of the vector (in a consistent sense) times the circumference of the loop.

1–5
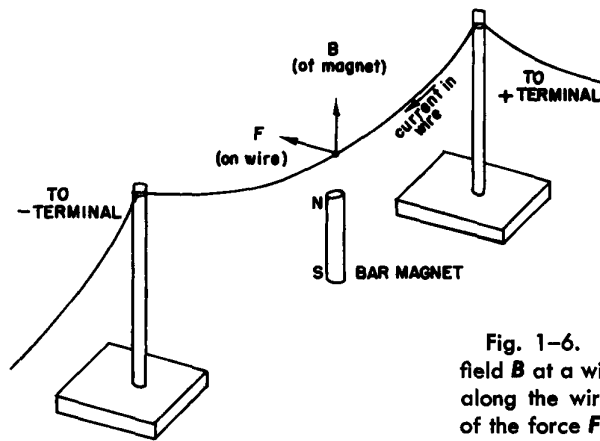
Fig. 1-6. A bar magnet gives a field **B** at a wire. When there is a current along the wire, the wire moves because of the force **F** = q**v** × **B**.

The constant c^2 that appears in Eq. (1.9) is the square of the velocity of light. It appears because magnetism is in reality a relativistic effect of electricity. The constant ϵ_0 has been stuck in to make the units of electric current come out in a convenient way.

Equations (1.6) through (1.9), together with Eq. (1.1), are all the laws of electrodynamics*. As you remember, the laws of Newton were very simple to write down, but they had a lot of complicated consequences and it took us a long time to learn about them all. These laws are not nearly as simple to write down, which means that the consequences are going to be more elaborate and it will take us quite a lot of time to figure them all out.

We can illustrate some of the laws of electrodynamics by a series of small experiments which show qualitatively the interrelationships of electric and magnetic fields. You have experienced the first term of Eq. (1.1) when combing your hair, so we won't show that one. The second part of Eq. (1.1) can be demonstrated by passing a current through a wire which hangs above a bar magnet, as shown in Fig. 1-6. The wire will move when a current is turned on because of the force **F** = q**v** × **B**. When a current exists, the charges inside the wire are moving, so they have a velocity **v**, and the magnetic field from the magnet exerts a force on them, which results in pushing the wire sideways.

When the wire is pushed to the left, we would expect that the magnet must feel a push to the right. (Otherwise we could put the whole thing on a wagon and have a propulsion system that didn't conserve momentum!) Although the force is too small to make movement of the bar magnet visible, a more sensitively supported magnet, like a compass needle, will show the movement.

How does the wire push on the magnet? The current in the wire produces a magnetic field of its own that exerts forces on the magnet. According to the last
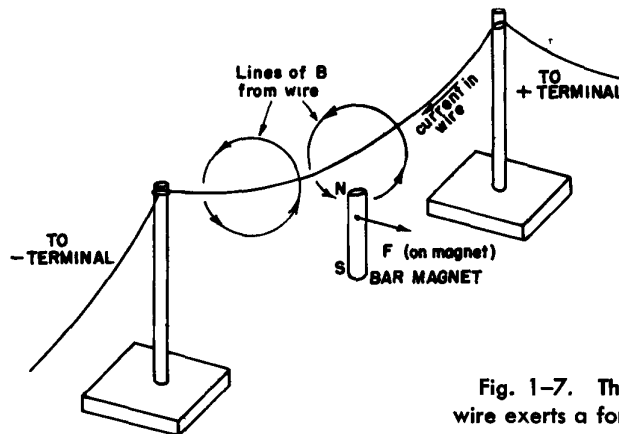
Fig. 1-7. The magnetic field of the wire exerts a force on the magnet.

* We need only to add a remark about some conventions for the *sign* of the circulation.

1-6
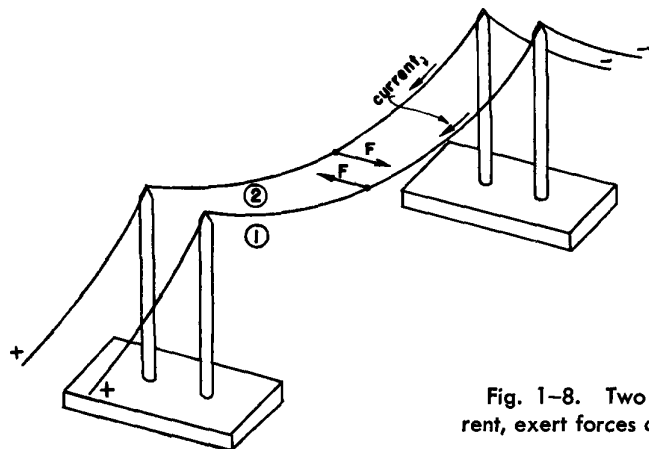
Fig. 1-8. Two wires, carrying current, exert forces on each other.

term in Eq. (1.9), a current must have a *circulation* of **B**—in this case, the lines of **B** are loops around the wire, as shown in Fig. 1-7. This **B**-field is responsible for the force on the magnet.

Equation (1.9) tells us that for a fixed current through the wire the circulation of **B** is the same for *any* curve that surrounds the wire. For curves—say circles—that are farther away from the wire, the circumference is larger, so the tangential component of **B** must decrease. You can see that we would, in fact, expect **B** to decrease linearly with the distance from a long straight wire.

Now, we have said that a current through a wire produces a magnetic field, and that when there is a magnetic field present there is a force on a wire carrying a current. Then we should also expect that if we make a magnetic field with a current in one wire, it should exert a force on another wire which also carries a current. This can be shown by using two hanging wires as shown in Fig. 1-8. When the currents are in the same direction, the two wires attract, but when the currents are opposite, they repel.

In short, electrical currents, as well as magnets, make magnetic fields. But wait, what is a magnet, anyway? If magnetic fields are produced by moving charges, is it not possible that the magnetic field from a piece of iron is really the result of currents? It appears to be so. We can replace the bar magnet of our experiment with a coil of wire, as shown in Fig. 1-9. When a current is passed through the coil—as well as through the straight wire above it—we observe a motion of the wire exactly as before, when we had a magnet instead of a coil. In other words, the current in the coil imitates a magnet. It appears, then, that a piece of iron acts as though it contains a perpetual circulating current. We can, in fact, understand magnets in terms of permanent currents in the atoms of the iron. The force on the magnet in Fig. 1-7 is due to the second term in Eq. (1.1).
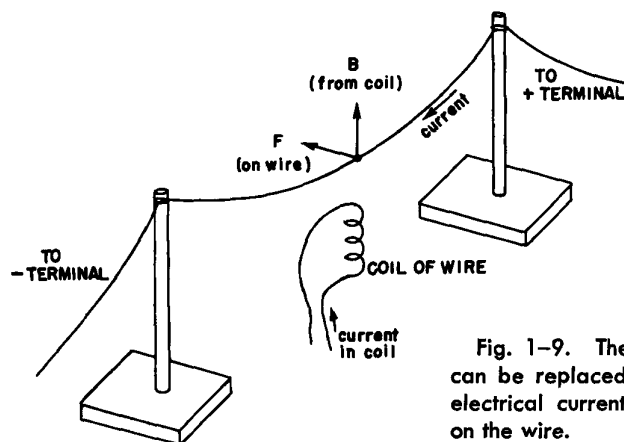
Fig. 1-9. The bar magnet of Fig. 1-6 can be replaced by a coil carrying an electrical current. A similar force acts on the wire.

1-7

Where do the currents come from? One possibility would be from the motion of the electrons in atomic orbits. Actually, that is not the case for iron, although it is for some materials. In addition to moving around in an atom, an electron also spins about on its own axis—something like the spin of the earth—and it is the current from this spin that gives the magnetic field in iron. (We say "something like the spin of the earth" because the question is so deep in quantum mechanics that the classical ideas do not really describe things too well.) In most substances, some electrons spin one way and some spin the other, so the magnetism cancels out, but in iron—for a mysterious reason which we will discuss later—many of the electrons are spinning with their axes lined up, and that is the source of the magnetism.

Since the fields of magnets are from currents, we do not have to add any extra term to Eqs. (1.8) or (1.9) to take care of magnets. We just take *all* currents, including the circulating currents of the spinning electrons, and then the law is right. You should also notice that Eq. (1.8) says that there are no magnetic "charges" analogous to the electrical charges appearing on the right side of Eq. (1.6). None has been found.

Fig. 1–10. The circulation of B around the curve C is given either by the current passing through the surface S_1, or by the rate of change of the flux of E through the surface S_2.

The first term on the right-hand side of Eq. (1.9) was discovered theoretically by Maxwell and is of great importance. It says that changing *electric* fields produce magnetic effects. In fact, without this term the equation would not make sense, because without it there could be no currents in circuits that are not complete loops. But such currents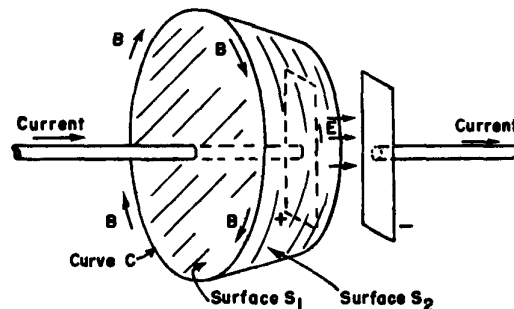 do exist, as we can see in the following example. Imagine a capacitor made of two flat plates. It is being charged by a current that flows toward one plate and away from the other, as shown in Fig. 1–10. We draw a curve C around one of the wires and fill it in with a surface which crosses the wire, as shown by the surface S_1 in the figure. According to Eq. (1.9), the circulation of B around C is given by the current in the wire (times c^2). But what if we fill in the curve with a *different* surface S_2, which is shaped like a bowl and passes between the plates of the capacitor, staying always away from the wire? There is certainly no current through this surface. But, surely, just changing the location of an imaginary surface is not going to change a real magnetic field! The circulation of B must be what it was before. The first term on the right-hand side of Eq. (1.9) does, indeed, combine with the second term to give the same result for the two surfaces S_1 and S_2. For S_2 the circulation of B is given in terms of the rate of change of the flux of E between the plates of the capacitor. And it works out that the changing E is related to the current in just the way required for Eq. (1.9) to be correct. Maxwell saw that it was needed, and he was the first to write the complete equation.

With the setup shown in Fig. 1–6 we can demonstrate another of the laws of electromagnetism. We disconnect the ends of the hanging wire from the battery and connect them to a galvanometer which tells us when there is a current through the wire. When we *push* the wire sideways through the magnetic field of the magnet, we observe a current. Such an effect is again just another consequence of Eq. (1.1)—the electrons in the wire feel the force $F = qv \times B$. The electrons have a sidewise velocity because they move with the wire. This v with a vertical B from the magnet results in a force on the electrons directed *along* the wire, which starts the electrons moving toward the galvanometer.

Suppose, however, that we leave the wire alone and move the magnet. We guess from relativity that it should make no difference, and indeed, we observe a similar current in the galvanometer. How does the magnetic field produce forces on charges at rest? According to Eq. (1.1) there must be an electric field. A moving magnet must make an electric field. How that happens is said quantitatively by Eq. (1.7). This equation describes many phenomena of great practical interest, such as those that occur in electric generators and transformers.

The most remarkable consequence of our equations is that the combination of Eq. (1.7) and Eq. (1.9) contains the explanation of the radiation of electromagnetic effects over large distances. The reason is roughly something like this: suppose that somewhere we have a magnetic field which is increasing because, say, a current is turned on suddenly in a wire. Then by Eq. (1.7) there must be a circulation of an electric field. As the electric field builds up to produce its circulation, then according to Eq. (1.9) a magnetic circulation will be generated. But the building up of *this* magnetic field will produce a new circulation of the electric field, and so on. In this way fields work their way through space without the need of charges or currents except at their source. That is the way we *see* each other! It is all in the equations of the electromagnetic fields.

1-5 What are the fields?

We now make a few remarks on our way of looking at this subject. You may be saying: "All this business of fluxes and circulations is pretty abstract. There are electric fields at every point in space; then there are these 'laws.' But what is *actually* happening? Why can't you explain it, for instance, by whatever it *is* that goes between the charges." Well, it depends on your prejudices. Many physicists used to say that direct action with nothing in between was inconceivable. (How could they find an idea inconceivable when it had already been conceived?) They would say: "Look, the only forces we know are the direct action of one piece of matter on another. It is impossible that there can be a force with nothing to transmit it." But what really happens when we study the "direct action" of one piece of matter right against another? We discover that it is not one piece right against the other; they are slightly separated, and there are electrical forces acting on a tiny scale. Thus we find that we are going to explain so-called direct-contact action in terms of the picture for electrical forces. It is certainly not sensible to try to insist that an electrical force has to look like the old, familiar, muscular push or pull, when it will turn out that the muscular pushes and pulls are going to be interpreted as electrical forces! The only sensible question is what is the *most convenient* way to look at electrical effects. Some people prefer to represent them as the interaction at a distance of charges, and to use a complicated law. Others love the field lines. They draw field lines all the time, and feel that writing E's and B's is too abstract. The field lines, however, are only a crude way of describing a field, and it is very difficult to give the correct, quantitative laws directly in terms of field lines. Also, the ideas of the field lines do not contain the deepest principle of electrodynamics, which is the superposition principle. Even though we know how the field lines look for one set of charges and what the field lines look like for another set of charges, we don't get any idea about what the field line patterns will look like when both sets are present together. From the mathematical standpoint, on the other hand, superposition is easy—we simply add the two vectors. The field lines have some advantage in giving a vivid picture, but they also have some disadvantages. The direct interaction way of thinking has great advantages when thinking of electrical charges at rest, but has great disadvantages when dealing with charges in rapid motion.

The best way is to use the abstract field idea. That it is abstract is unfortunate, but necessary. The attempts to try to represent the electric field as the motion of some kind of gear wheels, or in terms of lines, or of stresses in some kind of material have used up more effort of physicists than it would have taken simply to get the right answers about electrodynamics. It is interesting that the correct equations for the behavior of light in crystals were worked out by McCullough in 1843. But

people said to him: "Yes, but there is no real material whose mechanical properties could possibly satisfy those equations, and since light is an oscillation that must vibrate in *something*, we cannot believe this abstract equation business." If people had been more open-minded, they might have believed in the right equations for the behavior of light a lot earlier than they did.

In the case of the magnetic field we can make the following point: Suppose that you finally succeeded in making up a picture of the magnetic field in terms of some kind of lines or of gear wheels running through space. Then you try to explain what happens to two charges moving in space, both at the same speed and parallel to each other. Because they are moving, they will behave like two currents and will have a magnetic field associated with them (like the currents in the wires of Fig. 1-8). An observer who was riding along with the two charges, however, would see both charges as stationary, and would say that there is *no* magnetic field. The "gear wheels" or "lines" disappear when you ride along with the object! All we have done is to invent a *new* problem. How can the gear wheels disappear?! The people who draw field lines are in a similar difficulty. Not only is it not possible to say whether the field lines move or do not move with charges—they may disappear completely in certain coordinate frames

What we are saying, then, is that magnetism is really a relativistic effect. In the case of the two charges we just considered, travelling parallel to each other, we would expect to have to make relativistic corrections to their motion, with terms of order v^2/c^2. These corrections must correspond to the magnetic force. But what about the force between the two wires in our experiment (Fig. 1-8). There the magnetic force is the *whole* force. It didn't look like a "relativistic correction." Also, if we estimate the velocities of the electrons in the wire (you can do this yourself), we find that their average speed along the wire is about 0.01 centimeter per second. So v^2/c^2 is about 10^{-25}. Surely a negligible "correction." But no! Although the magnetic force is, in this case, 10^{-25} of the "normal" electrical force between the moving electrons, remember that the "normal" electrical forces have disappeared because of the almost perfect balancing out—because the wires have the same number of protons as electrons. The balance is much more precise than one part in 10^{25}, and the small relativistic term which we call the magnetic force is the only term left. It becomes the dominant term.

It is the near-perfect cancellation of electrical effects which allowed relativity effects (that is, magnetism) to be studied and the correct equations—to order v^2/c^2—to be discovered, even though physicists didn't *know* that's what was happening. And that is why, when relativity was discovered, the electromagnetic laws didn't need to be changed. They—unlike mechanics—were already correct to a precision of v^2/c^2.

1-6 Electromagnetism in science and technology

Let us end this chapter by pointing out that among the many phenomena studied by the Greeks there were two very strange ones: that if you rubbed a piece of amber you could lift up little pieces of papyrus, and that there was a strange rock from the island of Magnesia which attracted iron. It is amazing to think that these were the only phenomena known to the Greeks in which the effects of electricity or magnetism were apparent. The reason that these were the only phenomena that appeared is due primarily to the fantastic precision of the balancing of charges that we mentioned earlier. Study by scientists who came after the Greeks uncovered one new phenomena after another that were really some aspect of these amber and/or lodestone effects. Now we realize that the phenomena of chemical interaction and, ultimately, of life itself are to be understood in terms of electromagnetism.

At the same time that an understanding of the subject of electromagnetism was being developed, technical possibilities that defied the imagination of the people that came before were appearing: it became possible to signal by telegraph over long distances, and to talk to another person miles away without any connections between, and to run huge power systems—a great water wheel, connected by

filaments over hundreds of miles to another engine that turns in response to the master wheel—many thousands of branching filaments—ten thousand engines in ten thousand places running the machines of industries and homes—all turning because of the knowledge of the laws of electromagnetism.

Today we are applying even more subtle effects. The electrical forces, enormous as they are, can also be very tiny, and we can control them and use them in very many ways. So delicate are our instruments that we can tell what a man is doing by the way he affects the electrons in a thin metal rod hundreds of miles away. All we need to do is to use the rod as an antenna for a television receiver!

From a long view of the history of mankind—seen from, say, ten thousand years from now—there can be little doubt that the most significant event of the 19th century will be judged as Maxwell's discovery of the laws of electrodynamics. The American Civil War will pale into provincial insignificance in comparison with this important scientific event of the same decade.